

Problem Solving and Reasoning Progression

support materials to develop mathematical fluency, reasoning and problem solving

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The national curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex **problems** over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and nonroutine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions.

Reception Expectations

	Skills
Problem Solving	Engage with mathematical activities and problems.
	Independently choose to scaffold thinking using concrete and pictorial representations, if required.
	Independently choose to represent thinking using concrete, pictorial or abstract representations, as appropriate.
	With support (classroom discussion, paired or guided work) find a starting point to break into a problem.
	Use trial and trial strategy.
	Use ideas gained from a trial to decide what to do next.
	With support find possibilities.
	With support (adult, peer) check work (e.g. look for other possibilities and errors).
	With support pattern spot and copy and continue a pattern (actions, objects, shapes and numbers).

g	Skills
keasonin	Describe
	Listen to others' descriptions
Ω c	

Year 1 Expectations

	Skills
	Engage with mathematical activities and problems, making links and moving between different representations
ing	(concrete, pictorial, abstract).
	Independently choose to scaffold thinking using concrete and pictorial representations, if required.
20	Independently choose to represent thinking using concrete, pictorial or abstract representations, as appropriate.
Problem So	Begin to independently find a starting point to break into a problem.
	Use trial and improvement strategy.
	Independently find possibilities.
	With support (adult, peer) check work (.e.g. look for other possibilities, repeats, missing answers and errors).
	Independently pattern spot and copy and continue a pattern (objects, shapes, numbers, spatial) predicting what will
	come next.
	With support, investigate statements.

60	Skills
nir	Describe and explain with reasons.
so	
lea	Listen to others' explanations and try to make sense of them.

Year 2 Expectations

	Skills
	Engage with mathematical activities and problems, making links and moving between different representations
Problem Solving	(concrete, pictorial, abstract).
	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.
	Independently choose to represent thinking using concrete, pictorial or abstract representations, as appropriate.
	Independently find a starting point to break into a problem.
	With support work systematically.
	Independently find possibilities.
	Independently check work (e.g. look for other possibilities, repeats, missing answers and errors).
	Pattern spot and predict what will come next in a pattern/sequence (numbers, shapes, spatial).
	With support, investigate statements and conjectures.

Reasoning	Skills
	Explain with reasons and beginning to use given sentence stems and connectives to expand.
	Listen to others' explanations, make sense of them and compare and evaluate.
	Begin to edit and improve their own and a peer's explanation.
	Investigate 'what if?' questions.

Year 3 Expectations

	Skills
	Engage with mathematical activities and problems, making links and moving between different representations
	(concrete, pictorial, abstract).
Problem Solving	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.
	Independently choose to represent thinking using concrete, pictorial or abstract representations, as appropriate.
	Independently find an efficient way to solve a range of problems.
	Independently work systematically.
	Independently find possibilities using patterns spotted to support.
	Independently check and improve work (e.g. look for other possibilities, repeats, missing answers, errors and ways to
	improve).
	Pattern spot and predict what will come next in a pattern/sequence (numbers, shape or spatial).
	Independently investigate conjectures and provide examples and counter-examples.
	When they have solved a problem, pose a similar problem for a peer.

Reasoning	Skills
	Provide a convinced argument.
	Reflect on others' convinced explanations and use this to improve their work.
	Edit and improve their own and a peer's convinced explanation.
	Investigate 'what if?' questions.
	Create 'what if?' questions.

Year 4 Expectations

	Skills
	Engage with mathematical activities and problems, making links and moving between different representations
	(concrete, pictorial, abstract).
ല	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.
Problem Solvir	Independently choose to represent thinking using concrete, pictorial or abstract representations, as appropriate.
	Make suggestions of ways to solve a range of problems.
	Develop and apply a systematic approach.
	Find and predict possibilities that match the context using patterns spotted to support.
	Independently check and improve work (e.g. look for other possibilities, repeats, missing answers, errors and ways to
	improve).
	Pattern spot and with support, express generalisations/rules in words.
	Make and investigate conjectures and provide examples and counter-examples.
	When they have solved a problem, pose a similar problem for a peer.

Reasoning	Skills
	Provide a clear, correct, logical justification and with support, express generalisation/rules formed in words.
	Reflect on others' justifications and use this to improve their work.
	Edit and improve their own and a peer's justification.
	Investigate 'what if?' questions.
	Create 'what if?' questions.

Year 5 Expectations

	Skills
	Engage with mathematical activities and problems, making links and moving between different representations
	(concrete, pictorial, abstract).
BL	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.
lvir	Independently choose to represent thinking using concrete, pictorial or abstract representations, as appropriate.
oblem Sol	Make suggestions of ways to solve a range of problems.
	Organise work from the outset, looking for ways to record and work systematically.
	Find and predict possibilities that match the context using patterns spotted to support.
	Independently check and improve work (e.g. look for other possibilities, repeats, missing answers, errors and ways to
Р	improve).
	Pattern spot and independently express generalisations/rules in words.
	Make and investigate conjectures and provide examples and counter-examples.
	When they have solved a problem, pose a similar problem for a peer.

Reasoning	Skills
	Provide a clear, correct, logical justification, expressing generalisation/rules in words.
	Reflect on others' justifications and use this to improve their work.
	Edit and improve their own and a peer's justification.
	Investigate 'what if?' questions.
	Create 'what if?' questions.

Year 6 Expectations

	Skills				
Problem Solving	Engage with mathematical activities and problems, making links and moving between different representations				
	(concrete, pictorial, abstract).				
	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.				
	Independently choose to represent thinking using concrete, pictorial or abstract representations, as appropriate.				
	Make suggestions of ways to solve a range of problems.				
	Organise work from the outset, looking for ways to record and work systematically.				
	Find and predict possibilities that match the context using patterns spotted to support.				
	Independently check and improve their work (e.g. look for other possibilities, repeats, missing answers, errors and ways				
	to improve).				
	Pattern spot and begin to express generalisations/proof using words and symbolic notation.				
	Make and investigate conjectures and provide examples and counter-examples.				
	When they have solved a problem, pose a similar problem for a peer.				

Reasoning	Skills			
	Provide proof of reasoning, expressing generalisations in words and symbolic notation.			
	Reflect on others' proof and use this to improve their own work.			
	Edit and improve their own and a peer's proof.			
	Investigate 'what if?' questions.			
	Create 'what if?' questions.			

Problem	Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Skills	Engage with mathematical activities and problems.	Engage with mathematical activities and problems, making links and moving between different representations (concrete, pictorial, abstract).	Engage with mathematical activities and problems, making links and moving between different representations (concrete, pictorial, abstract).	Engage with mathematical activities and problems, making links and moving between different representations (concrete, pictorial, abstract).	Engage with mathematical activities and problems, making links and moving between different representations (concrete, pictorial, abstract).	Engage with mathematical activities and problems, making links and moving between different representations (concrete, pictorial, abstract).	Engage with mathematical activities and problems, making links and moving between different representations (concrete, pictorial, abstract).
	Independently choose to scaffold thinking using concrete and pictorial representations, if required.	Independently choose to scaffold thinking using concrete and pictorial representations, if required.	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.	Independently choose to scaffold thinking using concrete, pictorial or abstract representations, if required.
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	With support (classroom discussion, paired or guided work) find a starting point to break into a problem.	Begin to independently find a starting point to break into a problem.	Independently find a starting point to break into a problem.	Independently find an efficient way to solve a range of problems.	Make suggestions of ways to solve a range of problems.	Make suggestions of ways to solve a range of problems.	Make suggestions of ways to solve a range of problems.
	Use trial and trial strategy. Use ideas gained from a trial to decide what to do next.	Use trial and improvement strategy.	With support work systematically.	Independently work systematically.	Develop and apply a systematic approach.	Organise work from the outset, looking for ways to record and work systematically.	Organise work from the outset, looking for ways to record and work systematically.
	With support find possibilities.	Independently find possibilities.	Independently find possibilities.	Independently find possibilities using patterns spotted to support.	Find and predict possibilities that match the context using patterns spotted to support.	Find and predict possibilities that match the context using patterns spotted to support.	Find and predict possibilities that match the context using patterns spotted to support.
	With support (adult, peer) check work (e.g. look for other possibilities and errors).	With support (adult, peer) check work (.e.g. look for other possibilities, repeats, missing answers and errors).	Independently check work (e.g. look for other possibilities, repeats, missing answers and errors).	Independently check and improve work (e.g. look for other possibilities, repeats, missing answers, errors and ways to improve).	Independently check and improve work (e.g. look for other possibilities, repeats, missing answers, errors and ways to improve).	Independently check and improve work (e.g. look for other possibilities, repeats, missing answers, errors and ways to improve).	Independently check and improve their work (e.g. look for other possibilities, repeats, missing answers, errors and ways to improve).
	With support pattern spot and copy and continue a pattern (actions, objects, shapes and numbers).	Independently pattern spot and copy and continue a pattern (objects, shapes, numbers, spatial) predicting what will come next.	Pattern spot and predict what will come next in a pattern/sequence (numbers, shapes, spatial).	Pattern spot and predict what will come next in a pattern/sequence (numbers, shape or spatial).	Pattern spot and with support, express generalisations/rules in words.	Pattern spot and independently express generalisations/rules in words.	Pattern spot and begin to express generalisations/proof using words and symbolic notation.
		With support, investigate statements.	With support, investigate statements and conjectures.	Independently investigate conjectures and provide examples and counter- examples.	Make and investigate conjectures and provide examples and counter- examples.	Make and investigate conjectures and provide examples and counter- examples.	Make and investigate conjectures and provide examples and counter- examples.
				When they have solved a problem, pose a similar problem for a peer.	When they have solved a problem, pose a similar problem for a peer.	When they have solved a problem, pose a similar problem for a peer.	When they have solved a problem, pose a similar problem for a peer.

Reasoning	Reception	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Skills	Describe.	Describe and explain with reasons.	Explain with reasons and beginning to use given sentence stems and connectives to expand.	Provide a convinced argument.	Provide a clear, correct, logical justification and with support, express generalisation/rules formed in words.	Provide a clear, correct, logical justification, expressing generalisation/rules in words.	Provide proof of reasoning, expressing generalisations in words and symbolic notation.
	Listen to others' descriptions.	Listen to others' explanations and try to make sense of them.	Listen to others' explanations, make sense of them and compare and evaluate.	Reflect on others' convinced explanations and use this to improve their work.	Reflect on others' justifications and use this to improve their work.	Reflect on others' justifications and use this to improve their work.	Reflect on others' proof and use this to improve their own work.
			Begin to edit and improve their own and a peer's explanation.	Edit and improve their own and a peer's convinced explanation.	Edit and improve their own and a peer's justification.	Edit and improve their own and a peer's justification.	Edit and improve their own and a peer's proof.
			Investigate 'what if?' questions.	Investigate 'what if?' questions.	Investigate 'what if?' questions.	Investigate 'what if?' questions.	Investigate 'what if?' questions.
				questions.	questions.	questions.	questions.